Classical and Quantum Descriptions (*Wave-Particle Duality*)

At the macroscopic classical level, the position and momentum of a localized real particle system or wave packet may be described deterministically by ordinary time dependent differential equations or by a space-time dependent partial differential equation. At the microscopic quantum level, an accurate description of “particle” systems or wave packets dynamics requires use of a probabilistic analysis from which the average position and momentum of “particles” or wave packets may be inferred. The initial “position” of particle or wave packet systems must be described by a probabilistic wave function. The space-time evolution of this wave function follows from the requirement, at both the classical or quantum level, that the energy of the particle or wave packet systems is conserved. For an ensemble of particles or wave packets at a temperature T, the time independent equilibrium properties, at both classical or quantum levels, are describable by statistical thermodynamic methods.

Particle or Wave packet Descriptions

In an inertial reference frame moving with a constant velocity, small relative to light velocity, the classic Newtonian description of real particle or wave packet motion is a very good approximation if the space-time variability of the particle or wave packet is also small. If neither of these classical conditions is valid, a relativistic or quantum theoretic description is necessary. The classical equations of motion define the position $q(t)$ and momentum $p(t)$ at time t of a single real particle in p,q phase space**.** For motion in a gravitational or other type field, the energy Hamiltonian $H(p,q)$ (kinetic $p^2/2m$

plus potential $V(q)$), which is a constant of the motion, provides a convenient means for deriving the classical dynamical equations. A similar analysis exists for a localized wave packet of position $x(t)$ and wave momentum $k(t)$, with a dispersion relation $\omega(k,x)$ playing the role of the energy Hamiltonian.

For particle speeds that are a significant fraction of light speed, it is necessary to use Einstein's special theory of relativity, which constructs an amazing linkage of space and time and yields his famous kinetic energy $E=mc^2$ relation, m being a velocity dependent particle mass. In a local reference frame a particle moving at such speeds, a relativistic energy Hamiltonian $H(p,q)=mc^2+V(q)$ must be used to describe particle motion.

Classical single particle motion may also be described by a space-time function $\phi(q,t)$ that satisfies a wave equation, obtained by setting $-(\partial/\partial t)\phi = H(p,q)$ and $(\partial/\partial x)\phi = p$ whence one obtains the Hamilton-Jacobi equation:

$$-\frac{\partial\phi}{\partial t} = H(\frac{\partial\phi}{\partial q}, q),$$

or

$$\frac{\partial\phi}{\partial t} + \frac{1}{2m}\left(\frac{\partial\phi}{\partial q}\right)^2 + V(q) = 0\,.$$

The physical significance of this Hamilton-Jacobi equation is not clear classically. At the quantum level, one recalls that the Schroedinger wave function $\psi(q,t)$ describing the motion of a single mass particle is given by:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(\frac{\hbar\partial}{i\,\partial q}\right)^2 \psi + V(q)\psi$$

Seeking a solution of the form $\psi(q,t) = \exp[(i/\hbar)\phi(q,t)]$, $\hbar$ being Planck's constant of joule-sec dimension, one finds that

$$\frac{\partial\phi}{\partial t} + \frac{1}{2m}\left(\frac{\partial\phi}{\partial q}\right)^2 + V(q) - \frac{i\hbar}{2m}\frac{\partial^2\phi}{\partial q^2} = 0.$$

Thus in the limit $\hbar \to 0$, the meaning of $\phi(q,t)$ in the Hamilton-Jacobi equation becomes evident as the phase of the wave function $\psi(q,t)$.

Time dependent Statistical Wave Descriptions

For an ensemble of classical particles or wave packets described by an energy function *H(p,q)* in *p,q* phase space, a time-dependent ensemble kinetic distribution function *f(p,q,t)* provides a means for determining the average particle or wave packet position and momentum. A deterministic averaged kinetic equation for *f(p,q,t)* may be derived from a knowledge of *H(p,q)*; a more accurate statistical description requires taking random fluctuations into account. The overall ensemble may be described in terms of variables that are either kinetic (momentum, space, and time), fluid-dynamic (space and time), or macro-particle (time). The different descriptions are derivable from the kinetic by forming momentum or velocity and spatial moments, of the kinetic distribution function. Statistical thermodynamic descriptions of particle and wave packet ensembles in equilibrium at a temperature *T* also exist and usually are more general.

In the presence of classical randomness or in the microscopic quantum range, the description of single particle or wave packet motion requires a *statistical* space-time dependent wave treatment. In this range the probable average position <q(t)> of a particle or wave packet at time *t* may be defined in terms of a complex wave function $\psi(q,t)$, whose amplitude squared represents the probability density of finding the particle in q space at time t. The classical wave function $\psi(q,t)$ is governed by a wave equation that involves an operator energy Hamiltonian function $H(P,Q)$, P and Q being wave packet momentum and position operators in q-space. Derivation of the defining equation for $\psi(q,t)$ follows from the requirement that *averages* of particle dynamic observables, calculated from the probabilistic wave function $\psi(q,t)$, are identical to the averages of the classical particle or wave packet observables and obey the classical equations of motion and energy conservation. For relatively weak space-time wave variability, the wave description also reduces to the classical particle description. In this range, wave-particle duality well known in quantum mechanics is not unfamiliar in classical wave dynamics where it is usually expressed in ray optic terms.

Elaboration of the above views may be found in the following links:

Classical and Quantum Description of Particles and Wavepackets

Kinetic Quasiparticle Description of a Classical Wavepacket

Statistical Thermodynamics of Many-"Particle" Systems

Schroedinger Wave vs Kinetic Function Description of Many-"Particle" Systems